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STRESS WAVES IN GRANULAR MATERIAL

Final Report

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ABSTRACT

The results and findings of researches into stress wave propagation of granular media are presented. The purpose of this work is to clarify air-induced ground shock phenomena in soils, leading to reliable analytical methods of prediction, and to provide guidance for experimental verification and research.

This final report summarizes work on one-dimensional wave propagation of bi-linear, locking, and compacting materials, and presents the first results of the extension of these researches into two-dimensional wave phenomena in such non-linear media.

PUBLICATION REVIEW

This report has been reviewed and is approved.

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I. Introduction

This report is a summary outline of the work which has been accomplished under our current contract, AF29(601)-2855, the purpose of which was to investigate stress wave propagation phenomena in granular materials. Detailed results of this work have been issued in three technical reports:

> "Attenuation of Stress Waves in Bi-Linear Materials", by Richard Skalak and Paul Weidlinger, AFSWC-TN-60-30; October 1960.
>
> "A Method for Prediction of Ground Shock Phenomena in Soils" (Secret) by Paul Weidlinger and Alva Matthews, AFSWC-TDR-61-66; August 1961.
>
> "Step Load Moving with Low Subseismic Velocity on the Surface of a Half-Space of Granular Material", by Hans H. Bleich and Ewald Heer, AFSWC-TDR-63-2; April 1963.

This work is a continuation of a broad effort on this subject, and previous results have been published (see References 1, 5, 6).

This research effort in its narrower sense is directed towards the clarification of air-induced ground shock, but it also has a direct bearing on some other current investigations relating to the effect of direct ground shock.

Stress wave phenomena in granular materials open up an entirely new field of investigation where scarcity of both experimental and analytical results is prevalent. The earliest investigations by our group date back to 1958 (Reference 11) and to 1954 by USSR researchers (Reference 15).

The ultimate purpose of this work is to make available methods which will be suitable for the design of hardened underground construction; the specific purpose is to provide sufficient theoretical background for the performance and guidance of laboratory and field experiments.

In investigations which are restricted to one dimensional plane wave phenomena, a simple stress-strain relation will describe the behavior of the medium if strain rate effects can be neglected. Experimental evidence indicates that under one dimensional dynamic compression, granular materials show a hardening stress-strain relation; that is, the second derivative of the stress-strain curve is positive. This type of stress-strain curve has been approximated by various simplified models, such as locking materials and bi-linear materials. It was also shown that closed form or numerical solutions can be obtained for one dimensional waves in compacting medium, which has a more generalized form of stress-strain relation, and which is a good approximation of some experimentally obtained stress-strain curves.

As we proceed into investigation of cases where more than one spatial dimension is involved or where the wave front cannot be considered plane, information in addition to the stress-strain curve is required. The case of complete spherical symmetry which can be treated mathematically as a one dimensional case requires, for instance, the introduction of a yield condition which in granular materials can be derived from the assumption that Coulomb's law of failure is applicable under dynamic conditions. A separate law assuming a locking property of the medium must be postulated at the wave front.

As we investigate general two dimensional cases in addition to the yield condition, other properties of the medium must be postulated, such as flow rule, which can be expressed in relatively simple terms in the case of an isotropic material. It turns out that in addition to the flow rule, at least one further relation is required, which can be obtained by further postulating the existence of a plastic potential.

By means of these assumptions, the specific problem of stress waves generated by a step load moving with low subseismic velocity was considered and solved in our third report under the current contract. In the investigation of this problem, numerous assumptions regarding the behavior of the medium can be made which yield solutions which appear to be physically significant. Each of these solutions, which specifically consider whether the medium is compressible or incompressible, results in a slightly different behavior of the material. Further possible modes of response can be postulated as other cases and other velocities of the moving step load are considered. The plausibility of some of these assumptions can be supported by the conversion of the mathematical model into a simplified physical mechanism which in some respects can be considered analogous to the spring-dashpot model of standard solids. Nevertheless, the final choice between the various mathematically admissible modes of behavior must ultimately be resolved by laboratory and field experiments. Consequently, the analytical work itself is directed towards the solution of specific problems which are reproducible under laboratory conditions or in field experiments. The results so far obtained have shown that some of the quantitative results (such as attenuation of stresses and particle velocities) show close agreement with field measurements of weapons tests, and the existence of a dust precursor, (which was observed in weapons tests) has been demonstrated. These facts are important to the extent that they indicate that the basic assumptions are plausible and are supported by experimental evidence. On the other hand, because of the great difficulties which are inherent in laboratory work on granular materials, other significant phenomena can only be discovered (or distinguished from secondary effects and noise) with the aid and guidance of quantitative predictions based on the previously discussed theoretical analysis.

It is hoped that the current work will, in fact, lead to certain definite statements and clarification regarding the behavior of real materials through the selection of the particular mathematical model or mode of response, which corresponds to the results of experimental measurements. It is further anticipated that two-dimensional investigations will make it possible to conduct investigations into diffraction phenomena which are implicit in laboratory experiments due to the effect of measuring devices and the finiteness of the space in which the experiment is performed. Finally, the solution of the diffraction problem can lead to the determination of the effects of interaction of buried structures with stress waves in granular soils.

II. One Dimensional Plane Waves

One dimensional plane stress waves are characterized by a plane wave front with zero strain gradient parallel to the front and generally a non-zero gradient perpendicular to the front.

The medium ahead of the front is at rest and the stress is equal to zero. Behind the front the medium is in a state of plain strain. For this reason, the relevant physical characteristics of the medium are sufficiently defined by the initial density and the dynamic stress-strain law under uniaxial confined compression. The simplicity of the wave phenomena under these assumptions admits a variety of simple solutions which depend only on the choice of a few parameters defining the physical characteristics of the medium and that of the applied pressure pulse.

Such one dimensional plane waves are generated, if a semi-infinite medium is loaded by a pressure pulse of uniform intensity over the entire free boundary. One dimensional analysis will also be approximately valid if the free boundary is loaded by a pulse which advances over the surface with a velocity which is substantially greater than the propagation velocity in the medium. This type of loading is characteristic of the blast pressure generated by a nuclear explosion and consequently results derived from plane wave analysis can be used to provide information regarding the effects of air-induced ground shock in the superseismic range, which in many soils corresponds to peak ambient pressure above the 100 psi contours.

The simplified analysis also assumes that the soil is homogeneous and temperature effects, phase changes or transitions can be neglected. Subject to these limitations the results of analytical predictions are experimentally verifiable, provided that other relevant characteristics of the medium, i.e. stress-strain law and initial density are known.

1. Stress-Strain Characteristics of Granular Soil

The classical literature on soil mechanics provides extensive information regarding the static stress-strain characteristics of soils. More recently such experimental work has been extended to the dynamic behavior of fully confined soil samples (References 1, 2, 3, 9). These experiments generally seem to agree on the following points:

a) The material shows an initial elastic and possibly visco-elastic behavior at low stress levels and low density.

b) At higher stress levels large irrecoverable strains are manifested together with plastic behavior.

c) With increasing pressures a marked decrease of compressibility, i.e. a hardening in the stress-strain curve, is observed and unloading occurs on a nearly vertical branch with large irrecoverable strains accompanied by a significant increase in density.

d) Upon reloading, the material follows the unloading curve up to the original stress level and from there on shifts into the original loading branch of the diagram. At these high stress levels no significant strain rate effects are observed.

The typical form of such a stress-strain diagram is shown in Figure 1. It also should be noted that in some materials the initial elastic-plastic behavior is not observed and the effect of reduced compressibility (i.e. hardening) is the governing phenomenon. At the present state of the art no direct relationship between the shape of the stress-strain curve and other physical properties of the material, such as grain size, grain size distribution, cohesion, etc., is established.

2. Idealized Stress-Strain Relations

To allow an analytical treatment of one-dimensional soil dynamics problems, various stress-strain relations may be assumed. The validity and applicability of these simplified stress-strain laws depend on pressure levels and on the purpose for which these analytical models are to be used.

In the past several investigations have been conducted which are based on linear elastic behavior. Visco elastic behavior of a modified Voigt solid, represented by a spring dashpot model, was also considered (Reference 4).

The research which is reported under this contract is based on various non-linear materials of limited compressibility, the simplest of which is a stress-strain relation known as the ideal locking medium. (See Figure 2). The stress-strain law of the material is such that it exhibits no resistance up to a critical strain, which corresponds to the theoretical limit of its compressibility. Since the medium cannot be compressed beyond this limit the stress-strain diagram becomes vertical i.e. parallel to the stress axis both on loading and unloading. Other variations of this locking medium have also been considered which assume a different type of response before the critical strain is reached, such as an elastic behavior (Figure 3) or an elastic-plastic behavior (Figure 4). Stress wave phenomena in materials of these types have been treated in Reference 5. Spherical wave propagation has also been considered in Reference 6.

Another group of models proposes a bi-lineal stress-strain relation. In this medium, in its simplest form, no resistance up to a critical strain is assumed and, beyond this point, linear elastic response is postulated (Figure 5). A different assumption regarding the medium before it reaches the critical strain leads to a bi-linear material which shows an initial but smaller elastic resistance before the critical strain is reached (Figure 6). The properties of such media were analyzed in our first report TN60-30 under our current contract (Reference 7). By varying the elastic modulus of the second branch of the bi-linear model the entire domain of the linear elastic and the elastic-locking media is included (Figure 7). Wave propagation phenomena in the bi-linear model have been treated in Reference 7 for a discontinuous pressure input, i.e. a suddenly applied load. The formation and propagation of the strong shock for a continuous input have been treated recently in Reference 8.

A special case of bi-linear medium is also that of a material which exhibits linear stress-strain relation upon loading and follows a steeper, but also linear branch upon unloading. The unloading branch may become completely vertical approaching the behavior upon unloading of that of a locking material (see Reference 5 and Figure 8).

All the above-described stress-strain diagrams represent an attempt to approximate the actual behavior of granular soils, i.e. they represent an idealized model of the actual stress-strain diagrams shown in Figure 1.

Numerical calculations have shown that beyond a definable depth from the surface the decay of the peak stresses and particle velocities is not too sensitive to the general shape of the stress-strain curve as long as a general hardening, that is, limited compressibility, is assumed. In investigations which are directed toward free field phenomena at a great distance from the surface, the distinction between various types of bi-linear and/or locking behavior is only of limited importance (Reference 7).

In the second report, TDR61-66 under our current contract (Reference 9), an attempt has been made to consider a more general form of strain hardening. In this material the stress-strain curve bends towards the σ axis with increasing intensity and upon unloading shows irrecoverable strains at each stress level; that is, the unloading branch is assumed to be vertical. Specifically, it is assumed that the stress-strain curve is independent of strain rate effects. Upon loading it is characterized by

$$\frac{d^2\sigma}{d\epsilon^2} > 0$$

and upon unloading it follows the vertical line defined by

$$\epsilon = \epsilon_{max}$$

It reloads on the unloading curve up to the maximum stress and beyond that it follows the original loading curve. Since the medium is laterally confined, a constant and determinable lateral confining pressure exists, while zero lateral strain implies that the vertical strain is defined by

$$\epsilon = 1 - \frac{\rho_o}{\rho}$$

where ρ_o is the initial density (at $\sigma = 0$) and the density increases to ρ at the strain ϵ. It is further assumed that the material has no tensile strength. A medium which has these characteristics is called a compacting medium (Figure 9). The entire domain of such materials is bounded by two extreme forms of the stress-strain law. In one extreme we have the linear material shown in Figure 8 and in the other extreme the ideal locking medium shown on Figure 2.

3. One Dimensional Stress Waves in a Bi-Linear Material

Consider a semi-infinite space characterized by a stress-strain curve shown on Figure 5 which is subjected to a discontinuous pressure pulse over its free boundary. The shock velocity $\dot{z}$ will be

$$\dot{z}^2 = \frac{E_s}{\rho}$$

where E_s is the secant modulus corresponding to the stress level σ at the front. The one dimensional wave equations in Lagrangian coordinates are

$$\frac{\partial^2 \sigma}{\partial t^2} = c^2 \frac{\partial^2 \sigma}{\partial z^2}$$

$$\frac{\partial u^2}{\partial t^2} = c^2 \frac{\partial^2 u}{dz^2}$$

where $u(z,t)$ is the displacement of a particle at a time t from its initial position z, $c^2 = \frac{E_T}{\rho}$, and E_T is the tangent modulus of elasticity.

These wave equations can be solved by the numerical technique outlined in Reference 7 and they permit the evaluation of the influence of the parameters of the medium on the free field quantities. One of the significant findings is that beyond a predictable depth the peak stress and peak particle velocity of a bi-linear medium become nearly coincidental with that of an ideal locking medium of identical critical strain ϵ_c. This particular depth corresponds to the time at which the first and principal rarefaction wave from the surface arrives at the shock front. The practical implications of this are that the attenuation of stress waves in the far field depends principally upon the value of the critical strain ϵ_c and the decay time of the applied pulse.

4. <u>One Dimensional Stress Waves in a Compacting Medium</u>

Consider a semi-infinite space characterized by a stress-strain curve of the type shown in Figure 9, subjected to a discontinuous pressure pulse $p(t)$. The shock velocity of $\dot{z}$ will be

$$\dot{z}^2 = \frac{E_s}{\rho}$$

and the equation of motion is defined by

$$\frac{1}{2}\left(1 + \frac{E_s}{E_T}\right) \frac{d\sigma_z/dt}{\sigma_z} \int_0^t p\, dt = p - \sigma_z$$

where σ_z is the stress at the front.

We note that in the case of $E_s = E_T$ the above equation degenerates to that of the linear material of Figure 8 and in the case of $E_T = \infty$ to that of the ideal locking medium of Figure 2. The general form of the equation can be solved by numerical techniques, but a useful closed form solution is obtainable for the case

$$\frac{E_s}{E_T} = \text{constant}$$

which implies a parabolic stress-strain curve of the form

$$\sigma = \left(\frac{\epsilon}{k}\right)^n \quad .$$

where $k > 0$ and $n \geq 1$ are constant characteristics of the medium. The case $n = 1$ implies a linear medium and $n = \infty$ implies the locking medium. Particular values of n and k can be selected to approximate experimentally determined stress-strain curves. The intensity of the stress at the front is

$$\sigma_z = \frac{I^{\frac{2n}{n+1}}}{\frac{2n}{n+1} \int_0^t I^{\frac{n-1}{n+1}} \, dt}$$

where $I = \int_0^t p \, dt$ and the front location is given by

$$z = \frac{\left(\frac{2n}{n+1}\right)^{\frac{n+1}{2n}}}{(k\rho_o)^{1/2}} \left[\int_0^t I^{\frac{n-1}{n+1}} \, dt \right]^{\frac{n+1}{2n}}$$

The above equations define the attenuation of the peak stress with depth and permit the evaluation of all other free field quantities.

5. Application to the Prediction of Ground Shock Effects

Of the various one dimensional theories, that of the compacting medium turns out to be particularly suitable for the prediction of stress wave phenomena in soils. Currently under various contracts laboratory and HE field experiments are in progress in testing the applicability of this theory.

In our second report, TDR61-66 (Reference 9) it was shown that ground motion data obtained for nuclear field tests can be reproduced by the application of this theory and it was also shown that the prediction of air-induced ground shock effects corresponds fairly closely to most currently proposed empirical prediction procedures. The compacting medium theory seems also successful in defining shock spectra within frequency ranges of current practical interest.

In Table 1 below a comparison of quantities obtained by use of the compacting medium theory is shown with the semi-empirical methods proposed in Reference 10.

TABLE I

Comparison of Theory with ASCE Manual Predictions

Weapon: $W = 8$ MT $\quad P_o = 200$ psi

Soil: $w = 115$ #/ft^3 $\quad c = 2000$ ft/sec

Maximum Effects at Depth, in Feet

	Theory of this Paper			ASCE Manual Predictions		
Quantity	0'	50'	100'	0'	50'	100'
Peak Stress	200 psi	184 psi	169 psi	200 psi	180 psi	160 psi
Peak Particle Velocity						
Vertical	4.36 $\frac{ft}{sec}$	4.01 $\frac{ft}{sec}$	3.69 $\frac{ft}{sec}$	4.16 $\frac{ft}{sec}$	3.75 $\frac{ft}{sec}$	3.33 $\frac{ft}{sec}$
Horizontal	2.52 "	2.32 "	2.13 "	2.75 "	2.5 "	2.16 "
Displacement						
Vertical	12.3 in	10.9 in	9.6 in	14.5 in	13.3 in	12.0 in
Horizontal	7.1 "	6.3 "	5.5 "	4.8 "	4.4 "	4.0 "

III. Two Dimensional Problems of Wave Propagation

The knowledge of the effect of a surface pressure traveling with velocity V on the surface of an elastic half-space (Reference 13) has been one of the most useful results of the theory of elastic wave propagation. This is due to the fact that this problem in important regions represents good approximations for the effects of air blast from an atomic weapon in the vicinity of the shock front (Figure 10). The solution (13) has been a major tool in estimating pressure levels and obtaining shock spectra in locations at some distance from ground zero.

Obviously, the assumption of elastic behavior used in (13) is an oversimplification in case of soil, and the desirability of extending the theory to allow for the absorption of energy is quite apparent. The present studies are intended as an extension of (13), allowing for mechanisms of energy dissipation to be expected in granular materials.

Under the current contract, as reported in AFSWC-TDR-63-2 (Reference 12) a major research effort was made to introduce two types of dissipating mechanisms into the analysis, generalizing the problem treated in (13), in order to determine the effects due to a progressing load on the surface of a half-space for a dissipating material.

One type of material considered is controlled by internal friction of the Coulomb type. It is assumed that the material will behave linearly elastic, undergoing only small strains if the internal friction is not overcome. If, however, certain limiting states of stress are reached, internal slip will occur. Constitutive equations for a material according to this description, i.e. equations which give the relations between stresses and strains or strain rates, can be formulated on the basis of plastic potentials in the manner generally used in the analysis of elastic-plastic bodies. However, when forming the constitutive equations it was found that two types of formulations are possible. One, proposed in Reference 14, for static problems applies to a dilating material, i.e., to a material which increases in volume when internal slip occurs. The other formulation, previously suggested in (15), assumes no dilation due to slip. Both formulations are considered in our technical report (12).

In connection with the formulation of the boundary condition on a free surface of a granular medium, it was found necessary to consider the possibility of disintegration of the medium. The situation was explored on a mechanical model, simulating a medium with interior friction of the Coulomb type (Appendix B of 12). It was concluded that such disintegration, which has not been considered previously, occurs when a load moves with subseismic velocity on a half-space of a granular medium. Such disintegration leads to dust precursors ahead of the air shock, a matter which has indeed been observed.

The second type of material considered in which energy is dissipated is locking material. Work on one dimensional wave propagation, and for the case of spherical symmetry (which is in a sense also one dimensional), for such materials was discussed earlier in this report. The extension of the concept of locking behavior from one dimension to the case of two or three dimensions is a major step of a fundamental nature, and the result obtained under the contract is believed to be a significant achievement. The result concerns the state of stress and strain at the front of consolidation. In the one dimensional case of propagation of plane waves, the Rankine-Hugoniot equations give a relation between the stress and velocity of the consolidation front. This simple relation suffices to formulate the appropriate boundary value problem. In two or three dimensional cases, the situation is much more complicated, as the mechanism of locking in three dimensions must be introduced before the conditions at the locking front can be obtained.

Once the conditions on the consolidation front are understood, it is possible to treat two dimensional problems for locking materials which are elastic after locking or have other behavior. It is, therefore, possible to treat the locking material subject to Coulomb-slip after consolidation; i.e., the case treated in (12) can be combined with locking behavior.

1. Results Obtained for Granular Materials Subject to Internal Coulomb Friction

For this material, the case of a step pressure p progressing with a velocity V on the surface of a half-space, Figure 11, was considered. Depending on the state of stress, the material described above will behave elastically without slip, or its strains will be a combination of slip and elastic effects, such that the solution will be a combination of regions with slip, and without slip. To simplify the approach, and as a first step towards a more complete solution, the assumption was made that in regions where slip occurs, slip strains are appreciably larger than elastic strains, such that the latter can be neglected. Based on this assumption, closed form solutions for the problem were obtained, but, because of the above assumption, they are restricted to a subseismic range of the velocity V,

$$V \leq 0.2\ c_p \tag{1}$$

where $c_p^2 = \frac{E(1-\nu)}{\rho(1+\nu)(1-2\nu)}$ is the velocity of p-waves in the material, having the properties ρ, E, and ν, of the non-slip regions.

The character of the solution obtained can be described in terms of a system of polar coordinates r, Φ, the origin O of which coincides with the front of the applied pressure p, Figure 11. As indicated in this figure, there are three wedge shaped regions, separated by the lines $\Phi = 45^o$, $\Phi = 135^o$. Slip occurs only in one of them, $45^o < \Phi < 135^o$; in the two others the material acts elastically. The expressions for the stresses are given in (12). It is important that the vertical stress for $\Phi = 0$ (ahead of the pressure front p) does in general not vanish, but that in this location particles are expelled upwards due to disintegration of the material. The reactive pressure due to the expelled particles is the cause of the vertical stress just below the surface at $\Phi = 0$. As explained in detail in (12), surface disintegration occurs for all values of the applied load p if the material has no cohesion, but only for values $p > p_L$ if cohesion is present, where p_L is a limiting value depending on the value of cohesion.

The ultimate purpose of the analysis being the design of underground structures, the results of the analysis are best discussed on two pertinent effects. One quantity of interest is the total permanent strain at a point due to the passing of the shock wave on the surface. This strain gives an indication of the deformation to which a target, say a cylindrical shell, Figure 12, would be subjected if, in first approximation, the effect of the presence of the target on the free field is ignored. For a typical case, $k = 1/3$, the two permanent principal strains become

$$\epsilon_{1,2} = 0.111 \frac{p}{\rho V^2} \tag{2}$$

It is shown in (1, p. 35) that in the range of velocities given by Eq. (1), the strains (2) are appreciably larger than in an elastic material of similar properties where slip does not occur.

A second quantity of interest is the acceleration to which a target is subjected. The analysis indicates that the free field accelerations "a" are at each instant in a radial direction from point 0 at the head of the applied wave towards the target. The value "a" vanishes except while the slip region passes over the target. The time history of the acceleration is given in (1, Figure 20). Its maximum value, for $k = 1/3$, is

$$\max a \equiv 0.24 \frac{p}{\rho y} \tag{3}$$

where y is the depth of target. Again, it is demonstrated in (12) that the accelerations in the material with slip are an order of magnitude larger than in an elastic material of similar properties, but without slip.

It must be emphasized that the results obtained in (12) are not complete and further work is required. In order to obtain results valid for values of the velocity V above the limit in Eq. (1), particularly in the superseismic range, it is necessary to extend the analysis and include the presently neglected elastic effects in the slip range. This important extension is planned for a future contract. A second generalization, involving major mathematical difficulties concerns the treatment of decaying applied pressures, instead of step pressures. In the elastic case, the differential equations are linear, such that the treatment of decaying pulses is easily accomplished by superposition of the solutions (13) for a line load. In the present case, the differential equations are non-linear, such that the decaying pressure wave requires a novel analysis. An approach to this problem has been conceived, and a solution will be attempted in the future contract.

2. Results Obtained Concerning Two Dimensional Waves in Locking Media

The mathematical conditions which must be satisfied on the locking front for the case of one dimensional wave propagation, Figure 13, are known for a variety of types of material behavior after locking, i.e. rigid or linear elastic. These conditions are derived from momentum considerations and from the appropriate one dimensional stress-strain law in a relatively simple manner, because the principal stress and the particle velocities are in the one dimensional case necessarily in the direction of wave propagation, i.e. at right angles to the locking front.

The situation in case of two dimensional wave propagation, Figure 14, is much more complex. Considering a small element ds of the consolidation front, Figure 15, there may be a normal stress, σ, and a tangential one, τ. Correspondingly, the velocity of the particles at the front may have normal and tangential components. Using momentum considerations and the appropriate stress-strain law for plane strain, relations between stresses, particle velocities, and the velocity of propagation of the locking front can be formulated. However, the formulation becomes complicated because a locking material is a non-linear material, and the mathematical description of general non-linear relations between stress and strain in two and three dimensions requires sophisticated tools.

In spite of the complicated formulation required to state the problem, quite simple relations were finally obtained for isotropic locking materials. It was found that on a consolidation front, the tangential components τ of the stress must vanish, such that the normal stress σ, Figure 15, is the major principal stress. As a consequence, the particle velocity u is also normal to the front, quite similar to the case of one dimensional wave propagation. For example, when the locking strain ϵ is large compared to subsequent elastic or slip strains, one finds a relation which looks just as the result for one dimensional locking materials,

$$U^2 = \frac{\sigma}{\epsilon\rho} \tag{4}$$

where, however, σ is the compressive stress <u>normal</u> to the wave front, and U is the <u>normal</u> component of the velocity of the consolidation front.

In addition to finding that the shear stresses τ on the locking front vanish, such that the normal stress σ is a principal stress, the analysis also furnishes information on the tangential stresses and strains. It is found that the tangential strains vanish. In the case of a locking material which is linearly elastic after consolidation, the two principal stresses parallel to the locking front are therefore

$$\sigma_2 = \sigma_3 = \frac{\nu}{1-\nu}\sigma \qquad (5a)$$

Alternatively, if the material after locking is elastic, but subject to slip, the two principal stresses parallel to the locking front are

$$\sigma_2 = \sigma_3 = k\sigma \qquad (5b)$$

where the definition of k (and the behavior of the material in general) are as previously discussed in (12).

Once a relation between σ and U of the type of Eq. (4) is available, this relation, together with an equation of the type (5) for the principal stresses σ_2 and σ_3, permits the formulation of a boundary value problem for the region behind the locking front. As an example, the case of a step load of intensity p progressing with velocity V on the surface of a half-space has been tentatively considered for the case of a material which is linearly elastic after consolidation. It is found that there is a plane consolidation front making an angle β with the surface, Figure 16, where β depends on the values of p and V. The stresses, velocities, etc. in the wedge shaped region can be obtained from an elastic boundary value problem for this region. The known conditions on the horizontal surface and the relations (4) and (5a) on the consolidation front furnish the proper number of boundary conditions.

The details of these solutions, particularly for the case of material subject to Coulomb slip, will be considered in a future contract.

Conclusions

The researches summarized in this report show that air induced ground shock effects in the high superseismic range can be approximated by means of one dimensional wave propagation theory in a non-linear material of decreasing compressibility.

Predictions which are based on such one dimensional calculations show good agreement with ground motion measurements and also with various currently used semi-empirical methods.

Numerous additional problems which arise in ground motion predictions and in the evaluation of laboratory experiments require the solution of two dimensional phenomena. Under this contract, results were also obtained for a step pressure progressing with low subseismic velocity over the surface of a half space. Two distinct models are considered. In one case the material (depending on the state of stress) will respond elastically, but at other states slip will occur, subject to internal Coulomb friction. In the second case, the material exhibits locking characteristics and the propagation of a locking front is considered.

The results of these researches have led to the theoretical proof of the existence of dust precursors which have previously been observed but not explained in these terms. Future work on these topics should consider the use of a pressure pulse advancing with superseismic velocity over the surface of the half space.

REFERENCES

1. P. Weidlinger, "Analytical and Experimental Studies on Locking Media", 29th Symposium on Shock, Vibration and Associated Environment, Nov. 1960.

2. W. Heierli, "Die Dynamik eindimensionaler Bodenkörper im nichtlinearen, nichtelastischen Bereich", Doctoral Thesis E.T.H. Zurich, Switzerland, 1961.

3. A. A. Thompson, "A Comparison of the Dynamic and Static Stress-Strain Curve in Sand Under Confined and Unconfined Conditions", BRL Memorandum Report, No. 1262, April 1960.

4. N. M. Newmark, "Final Report of Special Studies Related to the WS107A-2 Launcher Installation", AMF, June 20, 1958.

5. M. Salvadori, R. Skalak and P. Weidlinger, "Waves and Shocks in Locking and Dissipative Media", Proceedings ASCE, Vol. 86, No. EM2, April 1960.

6. M. Salvadori, R. Skalak and P. Weidlinger, "Spherical Waves in a Plastic Locking Medium", Proceedings ASCE, Vol. 87, No. EM1, Feb. 1961.

7. R. Skalak and P. Weidlinger, "Attenuation of Stress Waves in Bi-Linear Materials", AFSWC-TN-60-30, Oct. 1960.

8. S. Sutcliffe, "Strong Shcok Formation in Bi-Linear Hardening Media", Proceedings ASCE, Vol. 89, No. EM2, April 1963.

9. P. Weidlinger and A. Matthews, "A Method for Prediction of Ground Shock Phenomena in Soils", AFSWC-TDR-61-66, Aug. 1961.

10. ASCE, "Design of Structures to Resist Nuclear Weapons Effects", ASCE Manual of Engineering Practice, No. 42.

11. M. Salvadori and P. Weidlinger, "Induced Ground Shock in Granular Media", STL, Bulletin Research Division, March 1958.

12. H. H. Bleich and E. Heer, "Step Load Moving with Low Subseismic Velocity on the Surface of a Half-Space of Granular Material", AFSWC-TDR-63-2, April 1963.

13. J. Cole and J. Huth, "Stresses Produced in a Half-Plane by Moving Loads", Journal of Applied Mech., Vol. 25, p.443, Dec. 1958.

14. D. C. Drucker and W. Prager, "Soil Mechanics and Plastic Analysis or Limit Design", Quart. Appl. Math., P. 157, 1952.

15. A. Y. Ishlinski, "On Two Dimensional Motion of Sand" (in Russian), Ukranian Math. Journal, Inst. of Math., Vol. VI, No. 4, 1954.

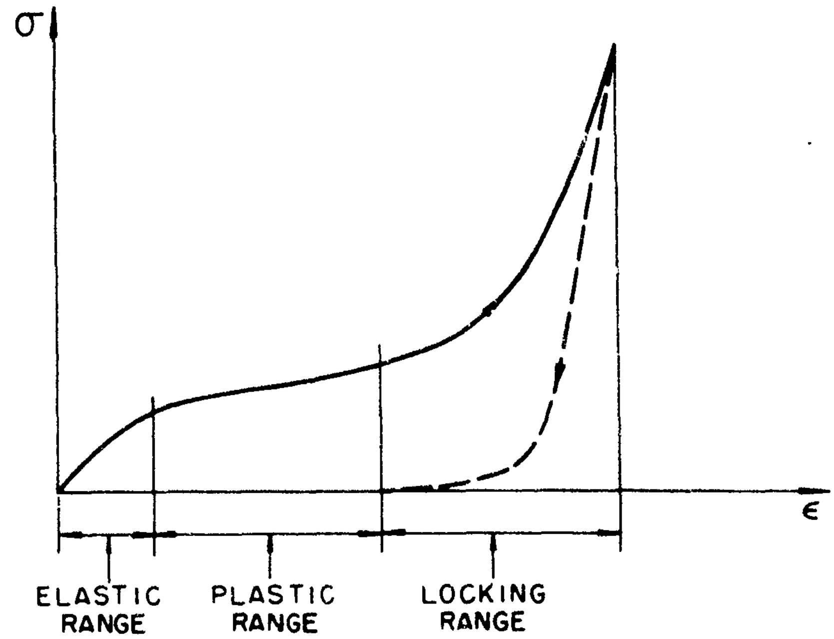


FIG. 1

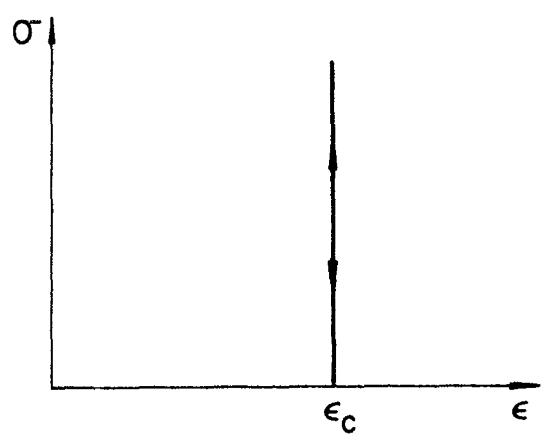
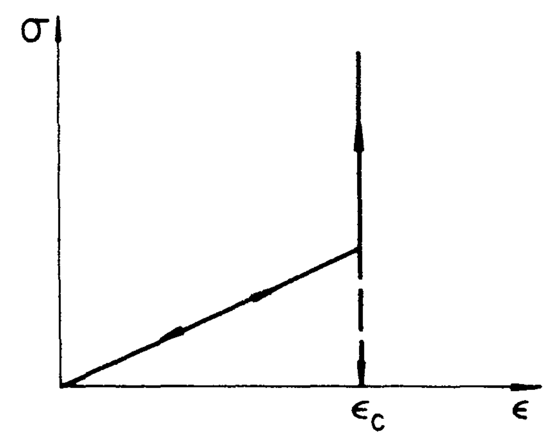


FIG. 2



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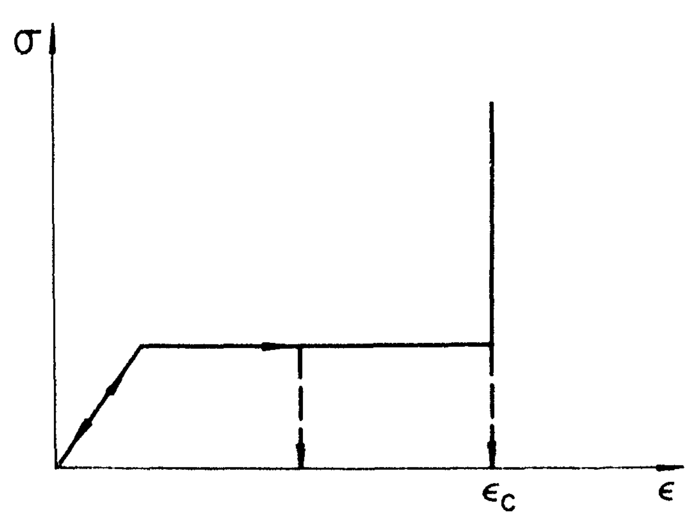

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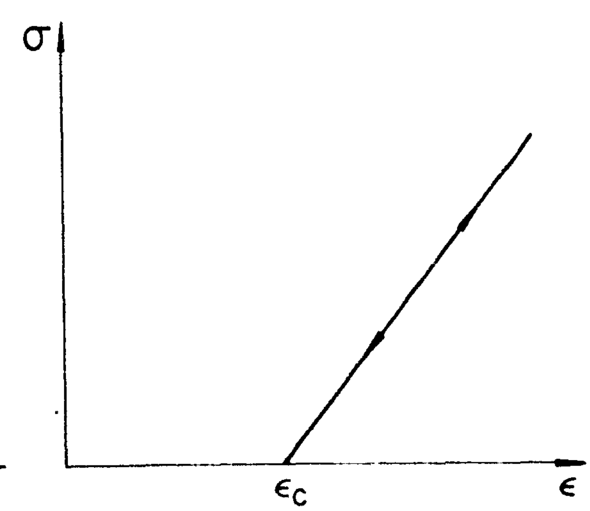

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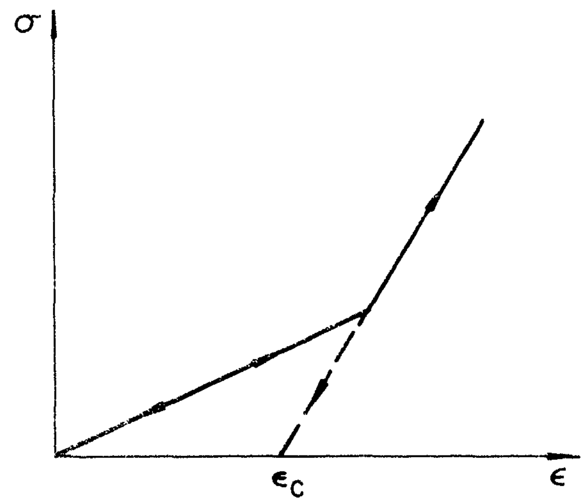
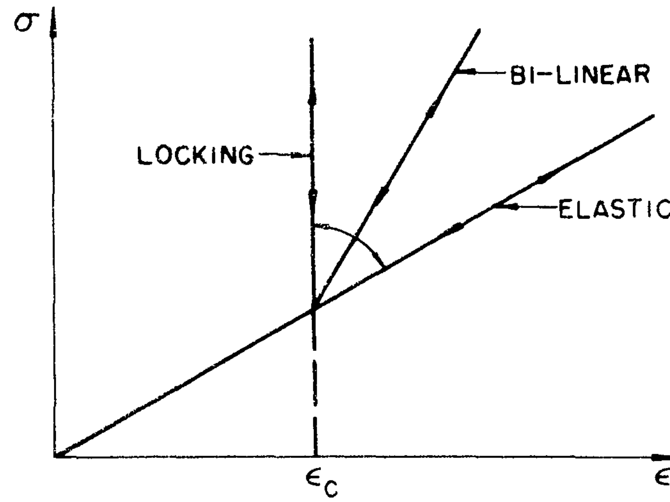


F I G . 6



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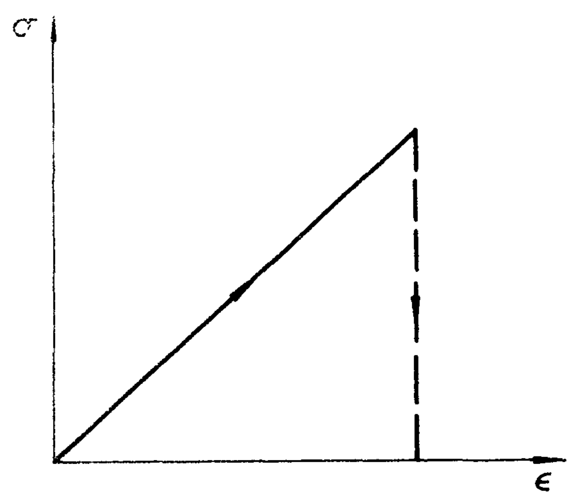

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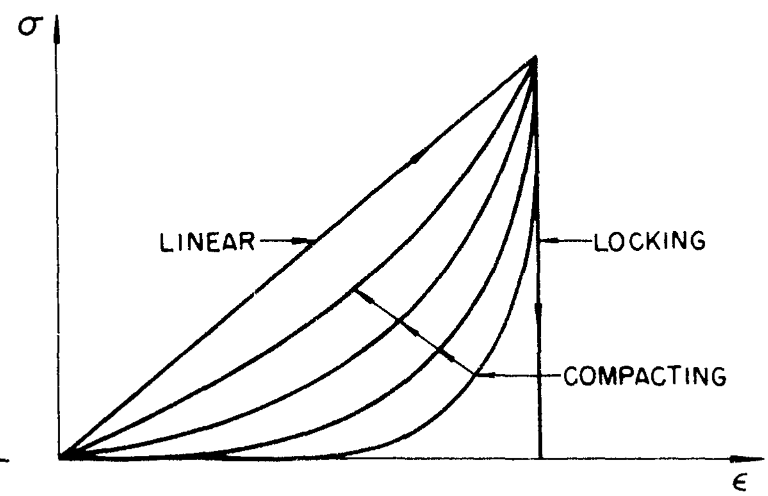

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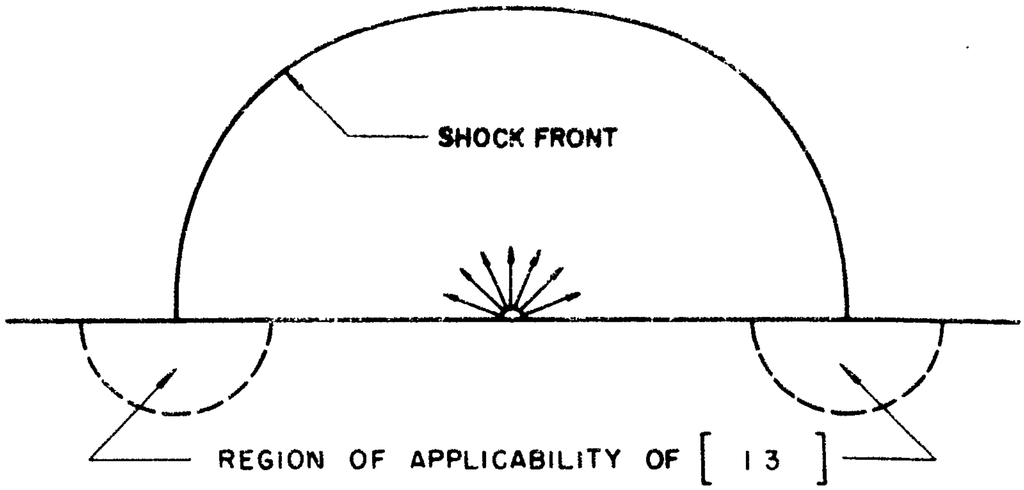


FIG. 10

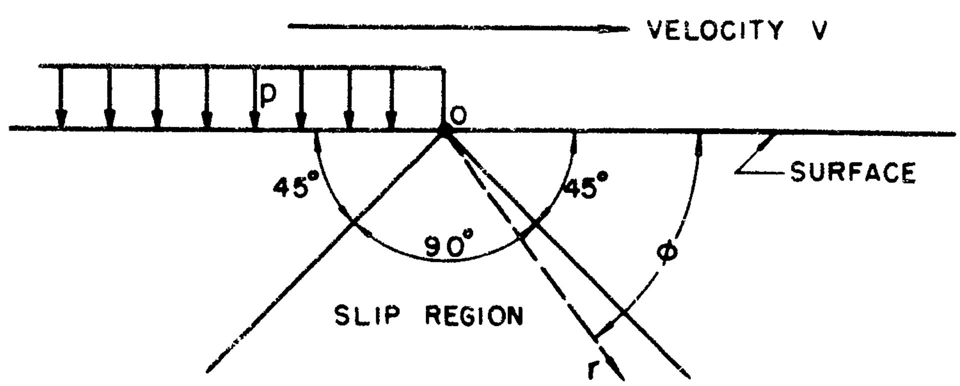


FIG. 11

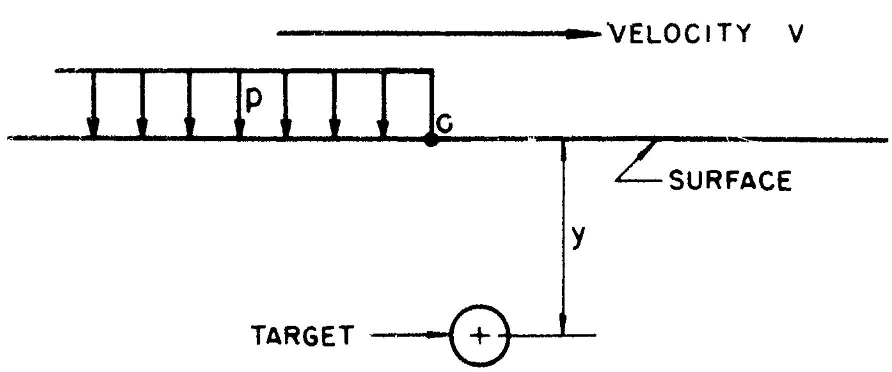
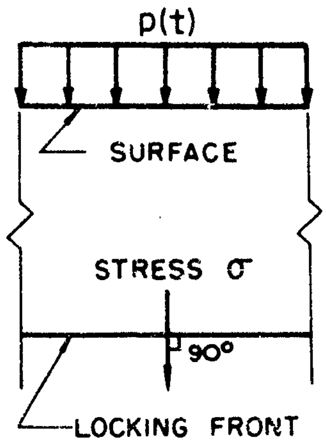
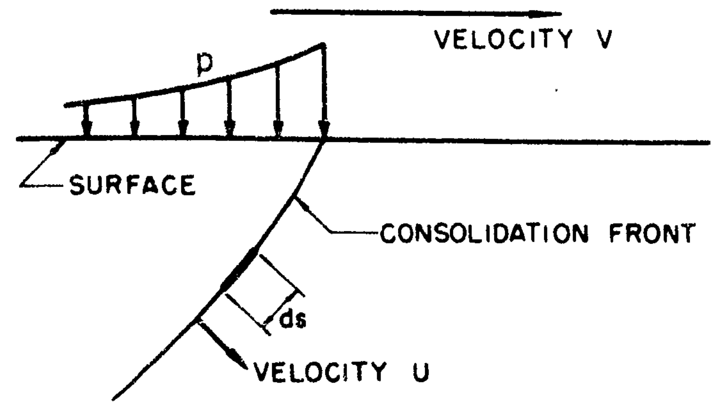


FIG. 12



F I G . 1 3



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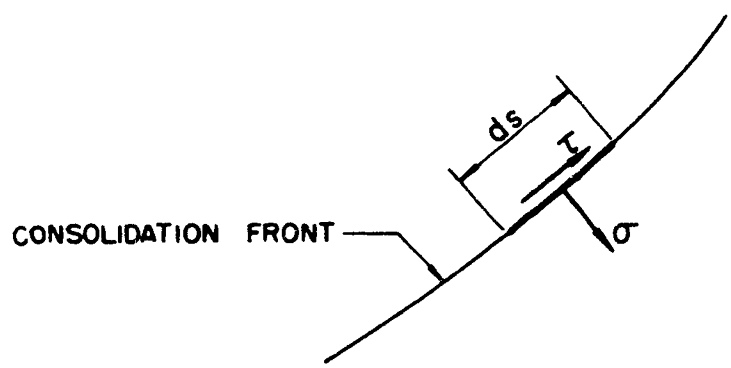

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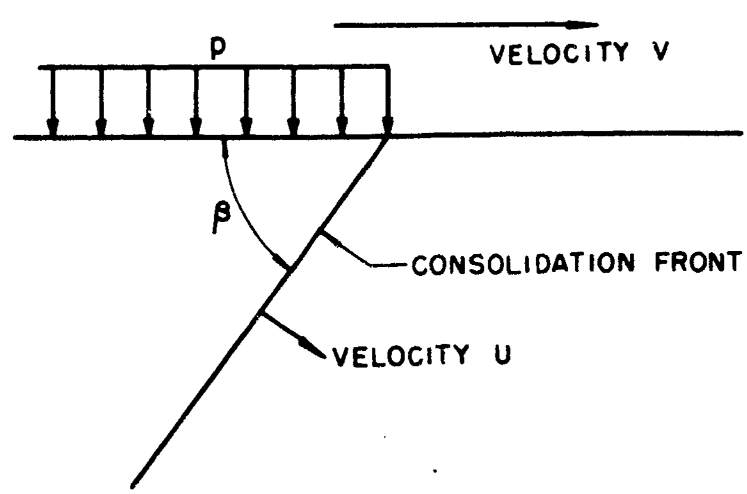

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